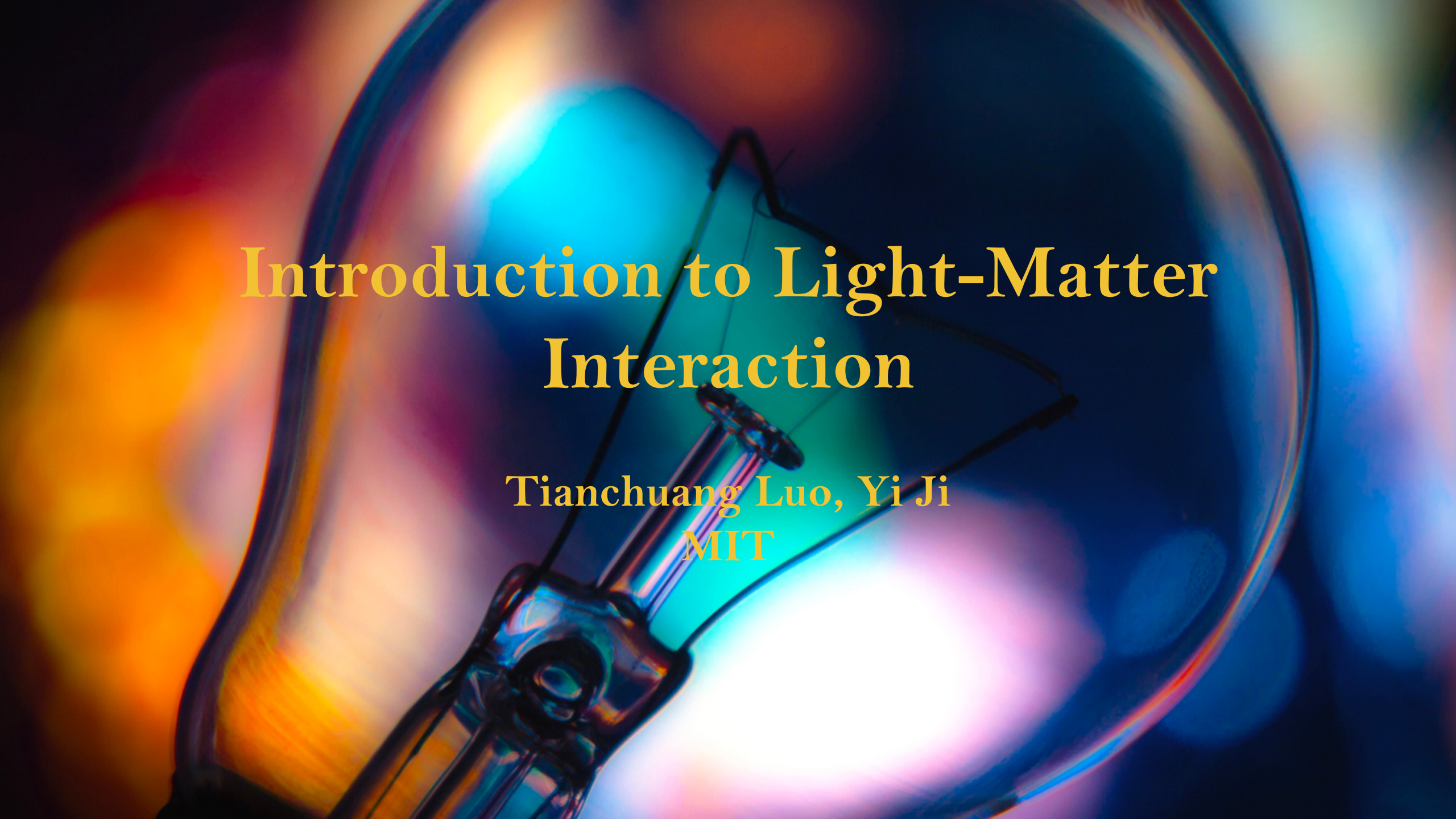

Introduction to Light-Matter Interaction

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Last time…

- Light can be seen as “waves”
  Wavelength λ, speed of light *c*, frequency ν $(\text{or } f)$, Amplitude

- Light can also be seen as “particles”
  Photon: unit energy $E = h\nu$

Exercise from last class

1. What is rough order of magnitude of photon emitted from a light bulb per second? On average what is the time for one photon to be emitted?

Assume $\lambda = 600\ \mathrm{nm}$
Photon energy $E = h\nu = 3.3\times10^{-19}\ \mathrm{J}$
Assume power of the light bulb $P = 10\ \mathrm{W}$
Number of photon emitted per second is $\frac{P}{E} = 3.0\times10^{19}\ \mathrm{s}^{-1}$

Exercise from last class

2. What is the absolute distance limit that you cannot see a signal light?

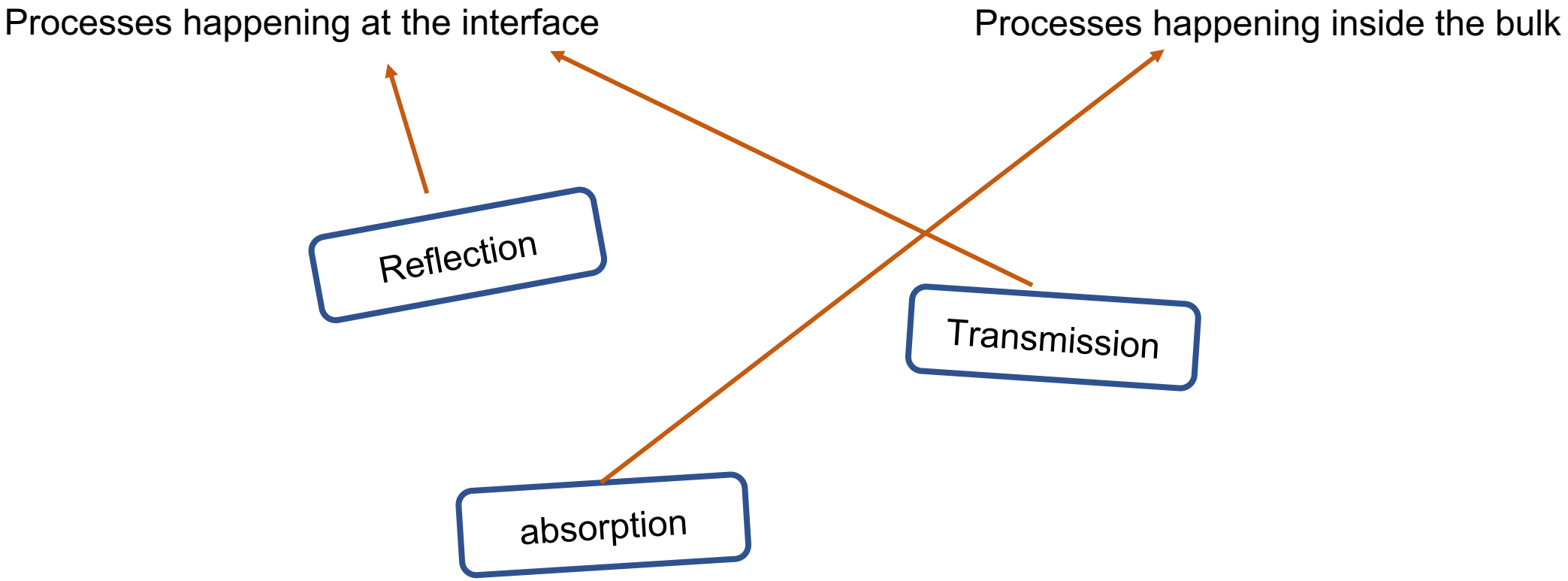
Assume signal power $P = 100\ \mathrm{W}$
Assume the area of human eyes $A_0 = 2\ \mathrm{cm}^2$
At distance L, the power received by an observer is given by $P \times \frac{A_0}{4\pi L^2}$
Assume wavelength $\lambda = 600\ \mathrm{nm}$
Number of photons captured by observer per second is $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{PA_0}{4\pi L^2} \times \frac{1}{h\nu} = \frac{PA_0\lambda}{4\pi hcL^2}$
Human eye will integrate signal in a certain time interval, which can be estimated by the inverse of movie frame rate (24fps): $t_0 = \frac{1}{24}\mathrm{s} \approx 0.04\ \mathrm{s}$.
The number of photon captured in this time interval is: $\frac{\mathrm{d}N}{\mathrm{d}t} \times t_0 = \frac{PA_0\lambda t_0}{4\pi hcL^2}$
For the signal to be ”seen”, this number has to be larger than 1:

$$\frac{\mathrm{d}N}{\mathrm{d}t} \times t_0 \geq 1$$

Which leads to $L \leq 1.3 \times 10^4\ \mathrm{km}$
The radius of Earth is 6371 km.

If we put $P = 4.5 \times 10^{26}\ \mathrm{W}$, which is the power output of the sun, we will have $L \leq 3100\ \mathrm{l.y.}$
The farthest star human naked eyes can see is 16,308 l.y. away.

How does light interact with matter?

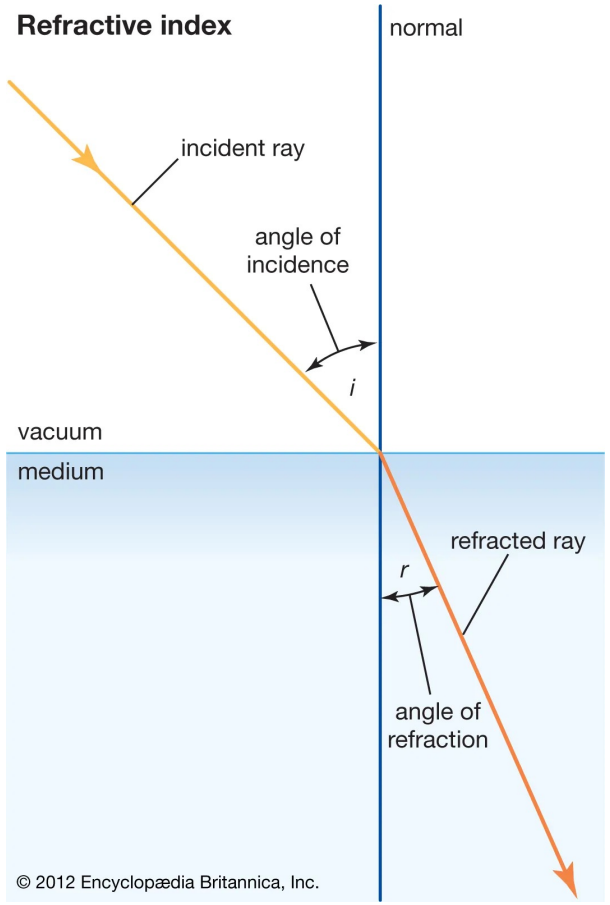


What describes light matter interaction?

- Refractive index

✓ Refractive index determines the diffraction angle at an interface:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



✓ Refractive index describes the change of speed of light in medium

$$c \to c/n$$
$$\lambda \to \frac{\lambda}{n}$$
$$\nu \to \nu$$

Light in air: $E = E_0 \cos\left(2\pi\left(\nu t - \frac{x}{\lambda}\right)\right) = E_0 \mathrm{Re}[e^{-2\pi i(\nu t - x/\lambda)}]$

Light in medium: $E' = E_0 \cos\left(2\pi\left(\nu t - \frac{x}{\lambda/n}\right)\right) = E_0 \mathrm{Re}[e^{-2\pi i(\nu t - nx/\lambda)}]$

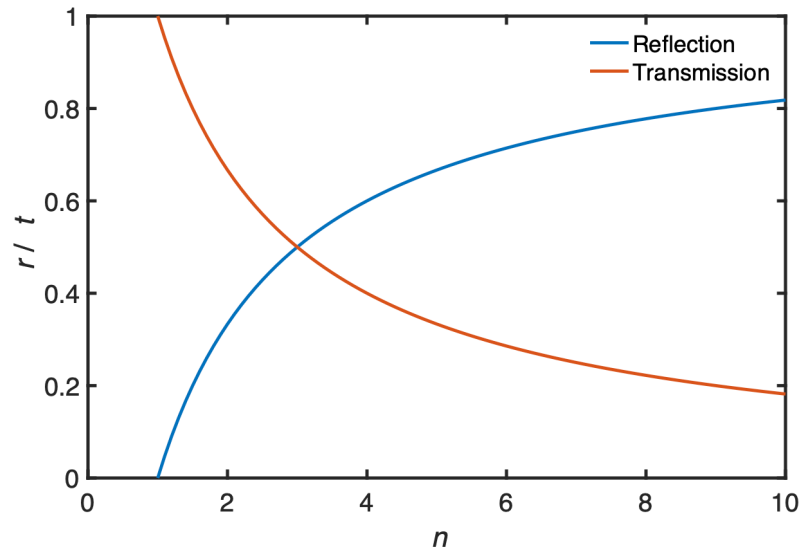
Reflectance and Transmittance

- ✓ Refractive index also determines how much light is transmitted and how much is reflected

- ➢ At normal incidence ($i = 0°$) between air and a material (refractive index n)

$$r = \frac{E_r}{E_i} = \frac{1-n}{1+n}$$

$$t = \frac{E_t}{E_i} = \frac{2}{1+n}$$



- ➢ Some common refractive index:
  - Water 1.33
  - Oil ~1.5
  - $CaCO_3$ ~1.65
  - Silver ~0.04
  - https://refractiveindex.info

- ➢ Other factors affecting r and t?

Questions

- Why doesn't the frequency of light change in medium?

- Why $r + t \neq 1$

- What if n is an imaginary value?

  $n \to n + i\kappa$

  Light in medium $E' = E_0 \operatorname{Re}\left[e^{-2\pi i\left(ft - \frac{nx}{\lambda}\right) - \frac{2\pi\kappa x}{\lambda}}\right] = E_0 \cos\left(2\pi\left(ft - \frac{x}{\lambda}\right)\right) e^{-2\pi\kappa x/\lambda}$

  Electromagnetic wave amplitude decays with the increase of x

  Absorption!

Refractive index is a function of light wavelength

The color of materials → selective reflection of light

Metals

diamond

Refractive index is a function of light wavelength

The color of materials → selective reflection of light

$$r = \frac{E_r}{E_i} = \frac{1-n}{1+n}$$

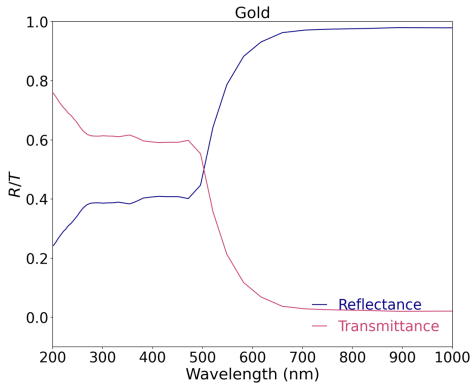
$$t = \frac{E_t}{E_i} = \frac{2}{1+n}$$

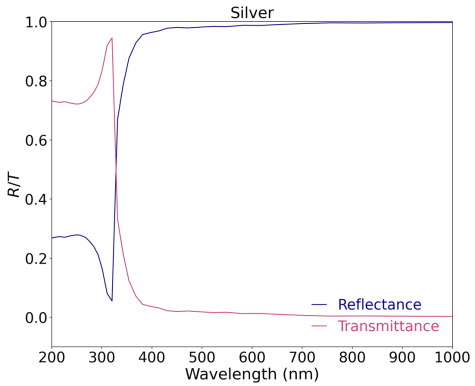
Metals

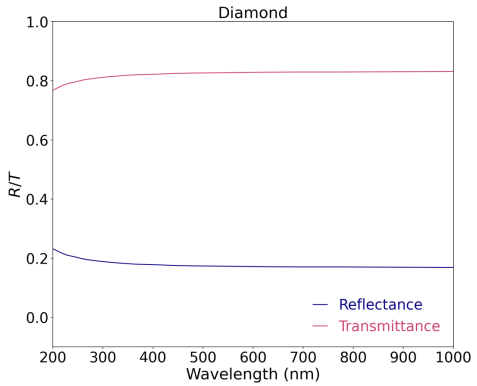
Gold

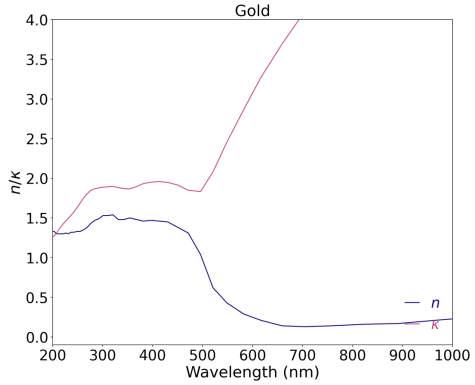
Silver

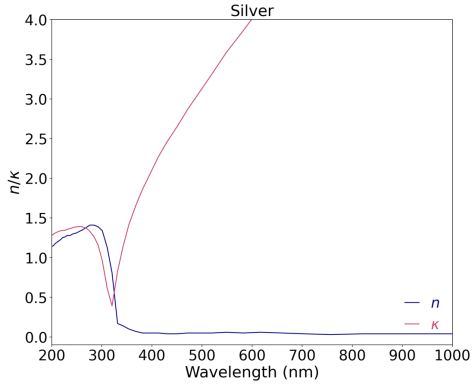
diamond

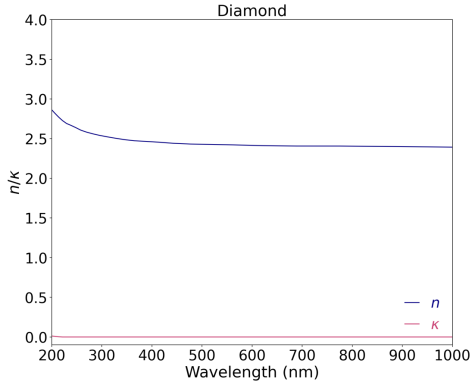






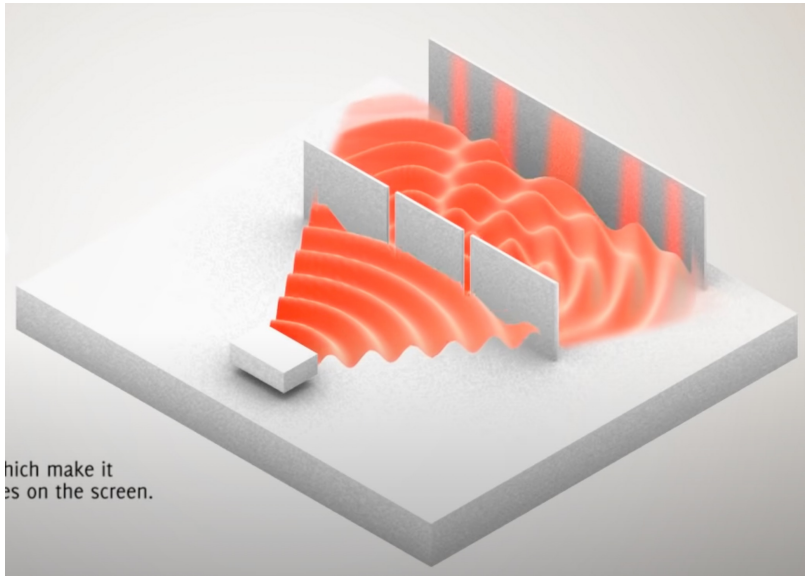


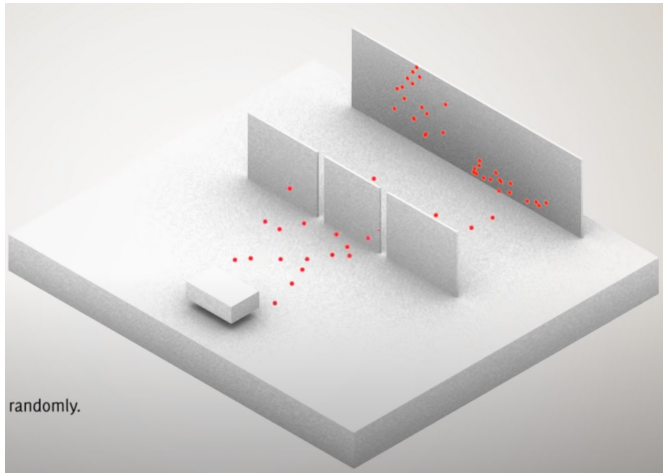


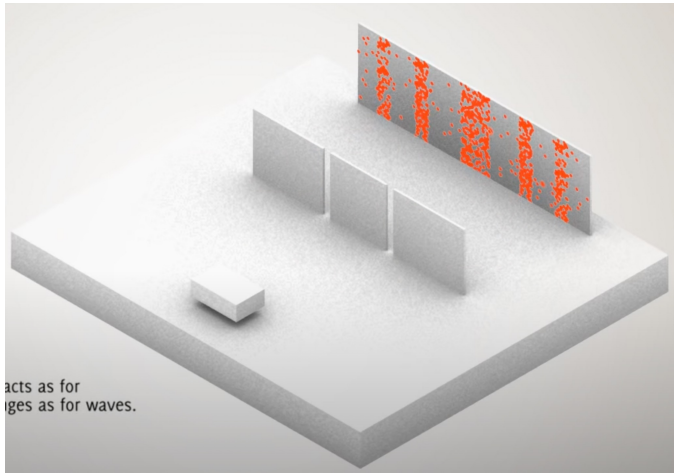


Wave – particle duality

Can particles be seen as waves?

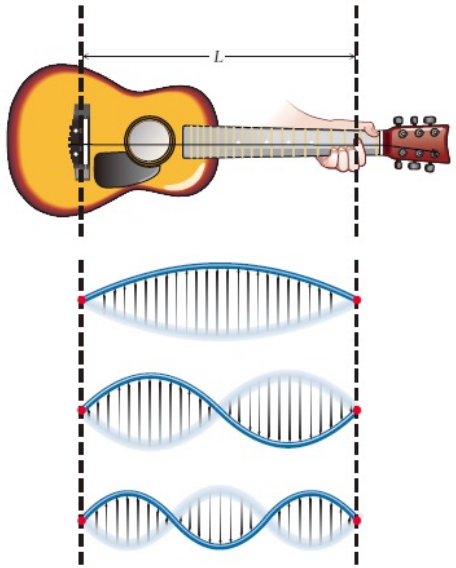


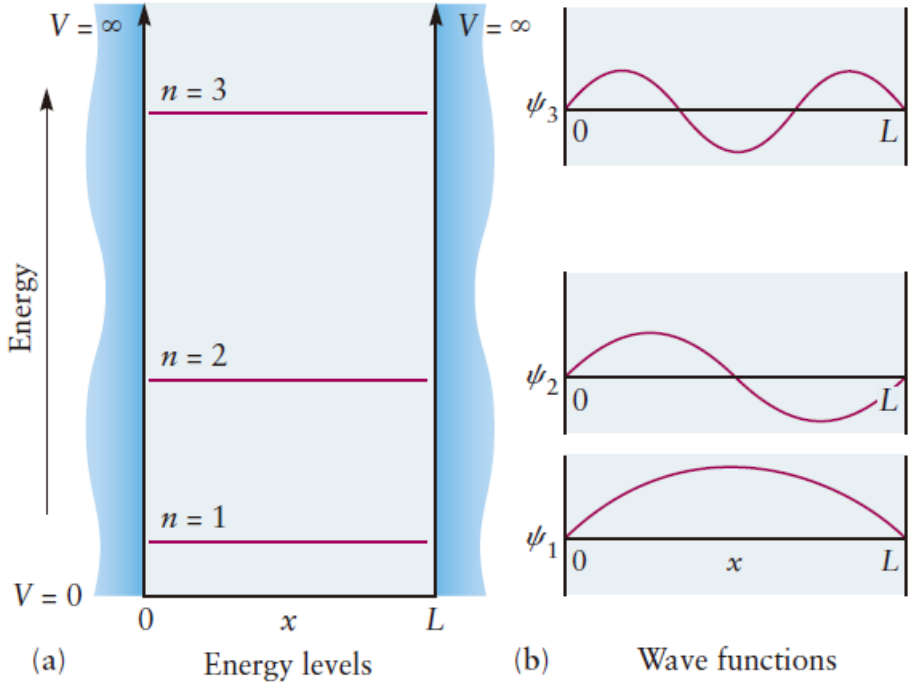




https://www.youtube.com/watch?v=qCmtegdqOOA

Particle in a box





- Standing waves

$$n\frac{\lambda}{2} = L \text{ or } \lambda = \frac{2L}{n} \qquad n = 1,2,3 \ldots$$

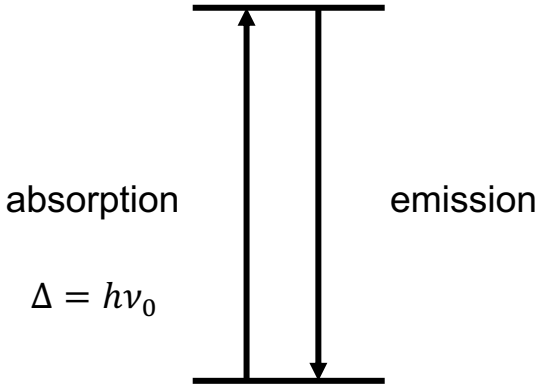
- Discrete energy levels

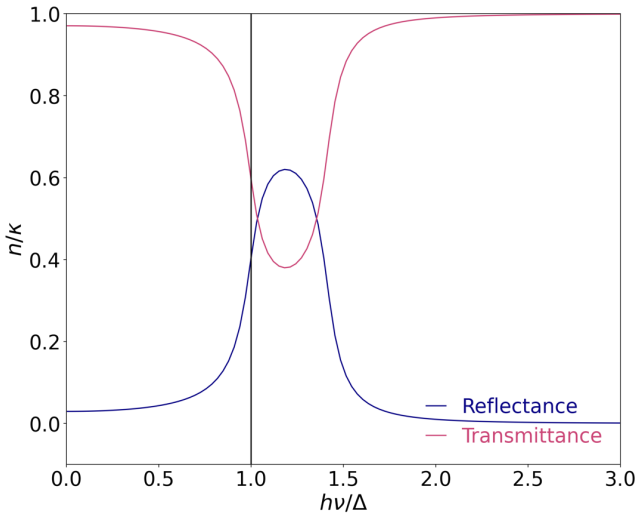
$$E_n = \frac{h^2 n^2}{8mL^2}$$

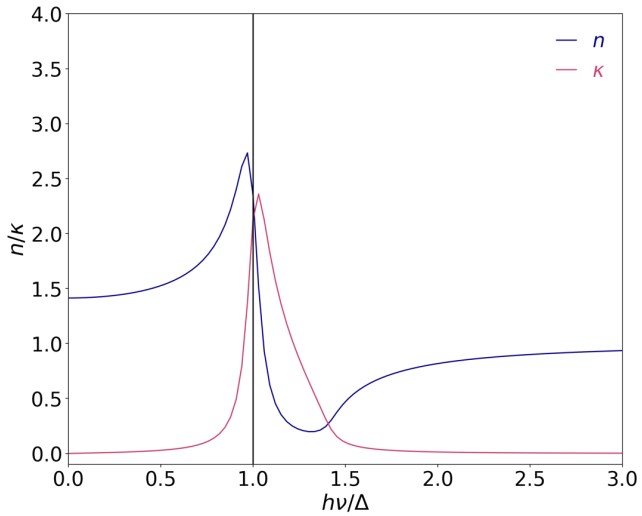
For macroscopic L, the discreteness is negligible.

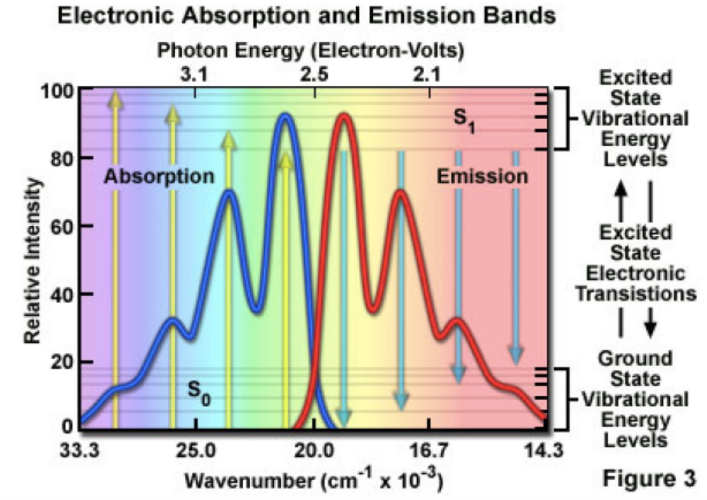
From Discrete energy levels to spectra

- Toy model: a two level system









Take home message

- Refractive index
  - ✓ Refraction angle, transmission, reflection
- Imaginary part of refractive index
  - ✓ Absorption

- Wave-particle duality
- Energy levels are discrete

Realistic spectrum of energy levels

Jablonski diagram